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ADHESIVE INTERACTION OF ELASTIC BODIES†

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The unified formulation of the axisymmetric problem of adhesion between elastic bodies and a method of solving it are presented. The adhesion is caused by either the surface energy of the bodies or menisci of fluid present in the contact zone. Both contact of the bodies and the case of the separated bodies are analysed. The values of the parameters for which the dependence of the load on the change of distance between the bodies becomes non-unique are determined. The loss of energy in an approach/separation cycle between the bodies as a function of the basic parameters of the problem is investigated analytically and numerically. © 2001 Elsevier Science Ltd. All rights reserved.

In the classical formulation of contact problems, the stresses are assumed to be compressive in the contact area and zero at the free surface of the interacting bodies. Thus, only repulsive forces between the surfaces are taken into account. The actual interaction potential between the solid surfaces has both a repulsive part and an attractive (adhesion) part, which is associated with the surface energy. Consideration of the attractive forces leads to tensile stresses at the boundary of the interacting bodies.

Contact problems for spherical elastic bodies, taking their surface energy into account, were formulated and solved analytically using various simplified forms of the interaction potential in [1–4]. A numerical analysis for the Lennard-Jones interaction potential in the exact form was carried out in [5]. It was shown that the relation between the force of interaction between the bodies and the distance between them is non-unique. This implies that the convergence of the bodies is irreversible and that a loss of energy occurs during the approach/separation cycle.

Not only solid bodies have surface energy but also the fluid films covering them. This can lead to the formation of menisci in the gap between the contacting bodies, which give rise to attraction of the surfaces – capillary adhesion. The contact of elastic bodies in the presence of capillary adhesion was considered in [6–8], and the distribution of the contact pressures, the size of the contact area, and other characteristics were calculated.

1. FORMULATION OF THE PROBLEM

Consider the interaction between two elastic bodies possessing surface energy. We will assume that the bodies are axisymmetric and the shape of their surface can be described by the power function

$$f(r) = f_1(r) + f_2(r) = Ar^{2n}$$

The bodies are pressed together by an external force q.

The gap $h(r)$ between the surfaces is given by

$$h(r) = f(r) + u(r) + d \tag{1.1}$$

where $i(r) = u_1(r) + u_2(r)$ is the total normal displacement of the surfaces of the interacting bodies due to their deformation, d is the change in the distance between two fixed points of the interacting bodies situated on the axis of symmetry of the bodies and far from the contact surface, as a result of the deformation of the bodies. If $h(0) > 0$, the surfaces of the bodies are not in contact and the value of d is positive. If $h(0) = 0$, the surfaces are in contact over an area $\Omega_c = \{r \leqslant a\}$ (Fig. 1), including point contact when $a = 0$. In this case, the value of d can be both positive and negative.

In order to take into account the surface energy of the bodies, we consider the area

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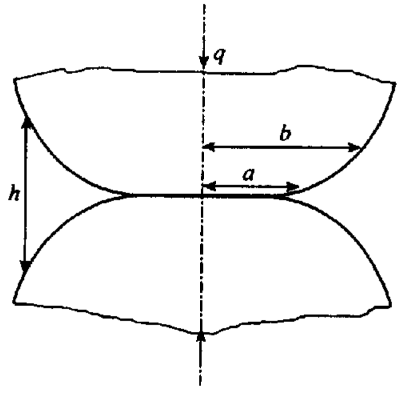

Fig. 1

$$\Omega_a = \begin{cases} 0 \leq r \leq b, & h(0) > 0 \\ a \leq r \leq b, & h(0) = 0 \end{cases} \tag{1.2}$$

where b is a certain distance such that $b \geq a$. We will assume that inside this area the surfaces are acted on by a constant negative pressure $-p_0$ caused by their surface energy.

As a result, we have a contact problem with the boundary conditions

$$p(r) = -p_0, \quad r \in \Omega_a; \quad h(r) = 0, \quad r \in \Omega_c \tag{1.3}$$

The last condition is the contact condition and occurs only if the surfaces are in contact with each other ($h(0) = 0$). If the surfaces are not in contact ($h(0) > 0$), the second condition of (1.3) is not included in the system of equations describing the problem.

The equilibrium condition

$$q = 2\pi \int_0^b rp(r)dr \tag{1.4}$$

is satisfied in all cases.

It is also necessary to specify the conditions for determining the quantities p_0 and b, which describe the adhesive interaction of the surfaces. Within the context of the problem, we can consider two forms of interaction between the bodies, taking the surface energy into account.

Adhesion of dry surfaces. Suppose the molecular interaction between the surfaces is described by the Lennard-Jones potential (Fig. 2, curve 1). Following the well-known method [1], we approximate this function by a step function (curve 2). The quantity $-p_0$ then has the meaning of the height of this step and the surface energy is defined by the relation

$$\gamma = \int_0^\infty p(z)dz = p_0 h_0$$

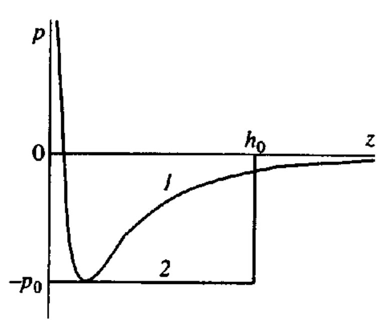

Fig. 2

whence we obtain the condition for determining b

$$h(b) = \gamma / p_0 \tag{1.5}$$

The parameters of the interaction potential, γ and p_0 are assumed to be given.

Capillary adhesion. Suppose there is a fluid in the gap between the bodies, which forms a meniscus occupying the area Ω_a. Then the uniform pressure p_0 applied to the surfaces in this area is the capillary pressure under the curved surface of the fluid, which is defined by Laplace's formula. If the wetting angle is zero and $b/l \leqslant 1 (l = (2A)^{-1/(2n-1)}$ is the characteristic size of the interacting bodies), the expression for the capillary pressure reduces to the form [8]

$$p_0 = 2\sigma / h(b) \tag{1.6}$$

where σ is the surface tension of the fluid. Introducing the notation $\gamma = 2\sigma$, we obtain an expression which is of the same form as (1.5). In this case, the value of p_0 is unknown. To determine it, we specify the volume of fluid υ in the meniscus. This volume is related to the geometry of the gap as follows

$$\upsilon = \int\int_{\Omega_a} rh(r)drd\varphi \tag{1.6}$$

Hence, to describe the adhesive interaction, we must specify γ and p_0 for dry surfaces and γ and υ when there is a meniscus.

To describe the elastic properties of the bodies, we will use the following relation between the normal displacements $u(r)$ of the surfaces and the normal pressures $p(r)$ which holds for axisymmetric contact of elastic bodies [9]

$$u(r) = \frac{4}{\pi E^*}\int_0^b p(r')\mathbf{K}\left(\frac{2\sqrt{rr'}}{r+r'}\right)\frac{r'dr'}{r+r'}, \quad 0 \leqslant r \leqslant b \tag{1.7}$$

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

where E_i and ν_i $(i = 1, 2)$ are Young's moduli and Poisson's ratios of the bodies, respectively and $\mathbf{K}(x)$ is the complete elliptic integral of the first kind.

Thus, the problem of the adhesion between the dry surfaces is defined by the system of equations (1.1)–(1.5) and (1.7). The problem of capillary adhesion is described by the system of equations (1.1)–(1.6) and (1.7).

2. DERIVATION OF THE BASIC RELATIONS

The case when there is no contact. The adhesive interaction between dry surfaces not in contact with each other is described by Eqs (1.1)–(1.5) and (1.7), excluding the last condition of (1.3). Substituting the first condition of (1.3) into relation (1.7) and taking the integral, we obtain [10]

$$u(\rho) = -\frac{4p_0 b}{\pi E^*}\begin{cases}\mathbf{E}(\rho), & \rho \leqslant 1 \\ \rho[\mathbf{E}(1/\rho) - (1 - 1/\rho^2)\mathbf{K}(1/\rho)], & \rho > 1; \ \rho = r/b\end{cases} \tag{2.1}$$

where $\mathbf{E}(x)$ is the complete elliptic integral of the second kind.

The value of the gap $h(r)$ is determined by substituting relation (2.1) for elastic displacements into relation (1.1). As a result, we obtain

$$h(r) = d - \frac{4p_0}{\pi E^*}bE\left(\frac{r}{b}\right) + Ar^{2n} \tag{2.2}$$

taking account into (2.2), we write relation (1.5) in the form

$$p_0\left(d - \frac{4p_0 b}{\pi E^*} + Ab^{2n}\right) = \gamma \tag{2.3}$$

Moreover, from the equilibrium equation (1.4), it follows that

$$q = -\pi b^2 p_0 \tag{2.4}$$

Relations (2.3) and (2.4) enable us to determine the force q applied to the bodies and the quantity d as functions of b, the radius of the area in which the adhesion pressure p_0 is applied. The solution obtained holds for $b \leq b^*$. The quantity b^* corresponds to the case where the surfaces are in contact at one point ($h(0) = 0$). From relation (2.2) when $h(0) = 0$ and relation (2.3) we obtain the equation for b^*

$$\frac{2(\pi-2)p_0 b}{\pi E^*} + Ab^{2n} = \frac{\gamma}{p_0} \tag{2.5}$$

In the case of capillary adhesion, we must supplement the relations obtained by the condition that the fluid volume (1.6) is constant and take into account the fact that the quantity p_0, describing the pressure in the fluid is not now specified but is one of the unknown quantities. Substituting expression (2.2) for the gap into condition (1.6) and taking the integral, we obtain

$$v = \pi d b^2 - \frac{16}{3E^*} p_0 b^3 + \frac{\pi}{n+1} A b^{2n+2} \tag{2.6}$$

Equations (2.3), (2.4), and (2.6) serve to determine the force q, the quantity d, and pressure in the fluid p_0, as a function of the radius b of the area occupied by the fluid when the surfaces are not in contact ($b < b^*$). To find the radius b^*, corresponding to point contact in the case of capillary adhesion, we need to solve the system of equations (2.5) and (2.6) for p_0 and b.

The case of contact between the surfaces. To solve the system of equations (1.1)–(1.5) and (1.7), we will use the method described previously [8]. As a result, we obtain the following relations:

For the force applied to the bodies

$$q = \frac{(2n)!!}{(2n+1)!!} 4E^* A n a^{2n+1} - 2p_0 b^2 (\varphi + c\sqrt{1-c^2}), \quad c = \frac{a}{b} \tag{2.7}$$

for the change in distance between the bodies,

$$d = -\frac{(2n)!!}{(2n-1)!!} A a^{2n} + \frac{2p_0 b}{E^*}\sqrt{1-c^2} \tag{2.8}$$

and also an equation relating the radius b of the contact area to the outer radius b of the area in which the adhesion pressure p_0 is applied, namely

$$\frac{2Aa^{2n}}{\pi}\left\{\left[\frac{(2n)!!}{(2n-1)!!} - \frac{1}{c^{2n}}\right]\varphi + \sqrt{\frac{1}{c^2}-1}\sum_{k=1}^{n}\frac{(2k-2)!!}{(2k-1)!!}c^{-2(n-k)}\right\}p_0 +$$

$$+\frac{4b}{\pi E^*}\{1 - c - \sqrt{1-c^2}\,\varphi\}p_0^2 + \gamma = 0, \quad \varphi = \arccos c \tag{2.9}$$

If the value of b is specified, Eq. (2.9) can be solved numerically for a, after which relations (2.7) and (2.8) enable one to determine the force q acting on the bodies and the quantity d.

The solution of the problem of capillary adhesion is also defined by relations (2.7)–(2.9) and, moreover, the condition (1.6) that the volume of fluid is constant. By means of the method described previously [8], this condition can be reduced to the form

$$v = 2Aa^{2n+2}\left\{\sqrt{\frac{1}{c^2}-1}\left[\frac{(2n)!!(2n-1)}{(2n+1)!!} + \frac{1}{n+1}\sum_{k=0}^{n}\frac{(2k)!!}{(2k+1)!!}c^{-2(n-k)}\right] -\right.$$

$$\left. -\frac{1}{c^2}\left[\frac{(2n)!!}{(2n-1)!!} - \frac{1}{c^{2n}}\frac{1}{n+1}\right]\varphi\right\} - \frac{4p_0 b^3}{3E^*}[4 - 3c - c^3 - 3\sqrt{1-c^2}\,\varphi] \tag{2.10}$$

Equations (2.9) and (2.10) serve to determine the radius a of the contact area and the pressure in the fluid p_0, provided that the outer radius b of the area occupied by the fluid is given. After this, the values of q and d are calculated from relations (2.7) and (2.8).

Note that Eq. (2.9), when $a = 0$, is identical with Eq. (2.5) for b^*, corresponding to point contact of the surfaces.

Thus, the solution of the problem of adhesion between dry surfaces is defined by relations (2.3) and (2.4) for the case when the surfaces are not in contact ($b < b^*$), and by relations (2.7)–(2.9) for surfaces in contact ($b > b^*$), the quantity b^* being bound from Eq. (2.5). The solution of the problem of capillary adhesion is given by relations (2.3), (2.4) and (2.6) for the case when the surfaces are not in contact, and relations (2.7)–(2.10) for surfaces in contact; the quantity b^*, corresponding to point contact, is found from Eqs (2.5) and (2.6).

3. ANALYSIS OF THE DEPENDENCE OF THE LOAD ON THE CHANGE IN THE DISTANCE BETWEEN THE BODIES

The relations obtained were used to analyse the dependence of the force q applied to the bodies on the quantity d, characterizing the change in the distance between them. For the adhesion of dry surfaces, we specify the parameters A and n, defining the shape of the bodies, the effective modulus of elasticity E^*, and the characteristics of the potential of molecular interaction, p_0 and γ. For capillary adhesion, we specify the values of A, n, E^*, and $\gamma = 2\sigma$, and also the volume v of the fluid in the meniscus.

Graphs of the dimensionless load $q/(p_0 l^2)$ ($l = (2A)^{-1/(2n-1)}$ is the characteristic size of the interacting surfaces) versus the dimensionless quantity d/l are presented in Fig. 3 for the adhesion of the dry surfaces for $n = 2$ and $E^*/p_0 = 1$. The thick lines correspond to contact between the surfaces, and the thin lines correspond to no contact. Curves 1 and 2 are constructed for $\gamma/(p_0 l) = 1$ and $\gamma/(p_0 l) = 2$, respectively. An analysis of the solution shows that the dependence of the load on the change in the distance between the bodies is non-unique for any values of the parameters. These results are similar to those obtained in [1] for the contact of a parabolic punch with an elastic half-space ($n = 1$).

Similar graphs of the dimensionless load $q/(E^* l^2)$ versus d/l are presented in Fig. 3(b) for capillary adhesion for $n = 2$ and $v/l_3 = 0.05$. Curves 1 and 2 correspond to the values $\gamma/(E^* l) = 0.05$ and $\gamma/(E^* l) = 0.1$, respectively. In this case, the curves are also non-unique, but only if the dimensionless surface tension $\gamma/(E^* l)$ is greater than a certain value.

The graphs in Fig. 3 enable us to analyse the processes of approach and separation for bodies possessing adhesion. Consider curve 1 in Fig. 3(a) and curve 2 in Fig. 3(b). If the load q is controlled (monotonically reduced), then at the point E corresponding to the minimum load $q_{\min}$, separation of the surfaces occurs suddenly. This effect occurs for any values of the parameters both for the adhesion of dry surfaces and for capillary adhesion. At the instant of separation, the surfaces are in contact over a finite area. If the quantity d is controlled (monotonically increased), the surfaces suddenly jump from point C to point D. When the quantity d is reduced, the surfaces jump from point A to point B. Note that points A and D always correspond to the absence of contact between the surfaces, whereas points B and C may correspond to both the contact and no contact. Thus, contact between the surfaces can be formed and broken suddenly. The sudden contact between the surfaces is illustrated in Fig. 4 for capillary adhesion between a punch and an elastic half-space.

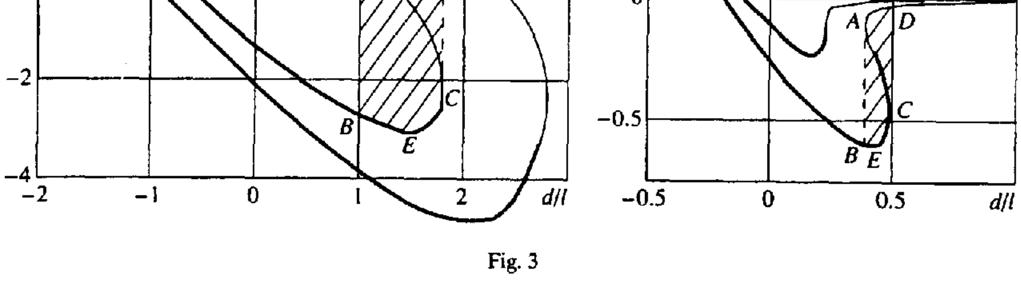

Fig. 3

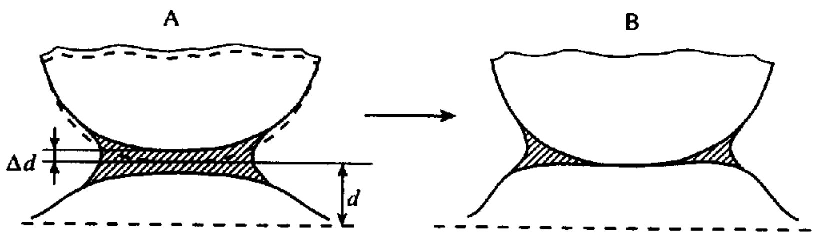

Fig. 4

Analysis of the approach/separation cycle shows that the work required to separate the surfaces is not equal to the work returned when they come together. The corresponding loss of energy is defined by the area of the hatched regions in Fig. 3:

$$\Delta w = \int_{ABCD} q(d)\,\mathrm{d}d \tag{3.1}$$

The dependence of the load on the change in the distance between the bodies enables us to investigate the loss of energy Δw and the pull-off force $q_{\min}$.

Graphs of the dimensionless loss of energy $\Delta w/(p_0 l^3)$ and the pull-off force $q^{\min}/(p_0 l^2)$ versus the dimensionless surface energy $\gamma/(p_0 l)$ are presented in Figs 5(a) and (b), respectively, for the adhesion of dry surfaces ($n = 1$, the solid curves). The results show that as the surface energy increases, the loss of energy increases and reaches a constant value at a certain value of $\gamma/(p_0 l)$. The loss of energy $\Delta w/(p_0 l^3)$ is greater for smaller values of E^*/p_0, i.e., for softer bodies. For comparison, graphs for another shape of the bodies are presented for $E^*/p_0 = 2$ (the dashed curves).

Graphs of the dimensionless loss of energy $\Delta w/(E^* l^3)$ versus the dimensionless surface tension $\gamma/(E^* l)$ are presented in Fig. 6 for the case of capillary adhesion ($n = 1$, the solid curves). The loss of energy is non-zero only if the dimensionless surface tension is greater than a certain value. As the dimensionless surface tension increases, the loss of energy increases without limit. Besides, the value of $\Delta w/(E^* l^3)$ is greater the smaller volume of fluid in the meniscus. A graph for another shape of the surfaces ($n = 2$) for $v/l^3 = 0.05$ is also shown (the dashed curve).

Parabolic bodies. The case when $n = 1$ corresponds to the interaction of surfaces of parabolic shape. In this case, the quantity l is the reduced radius of the surfaces, $l \equiv R = (R_1^{-1} + R_2)^{-1}$.

Analysis of the solution of the problem of the adhesive interaction between dry parabolic surfaces indicates that the relation between the dimensionless load Q and the dimensionless quantity D defined by the relation

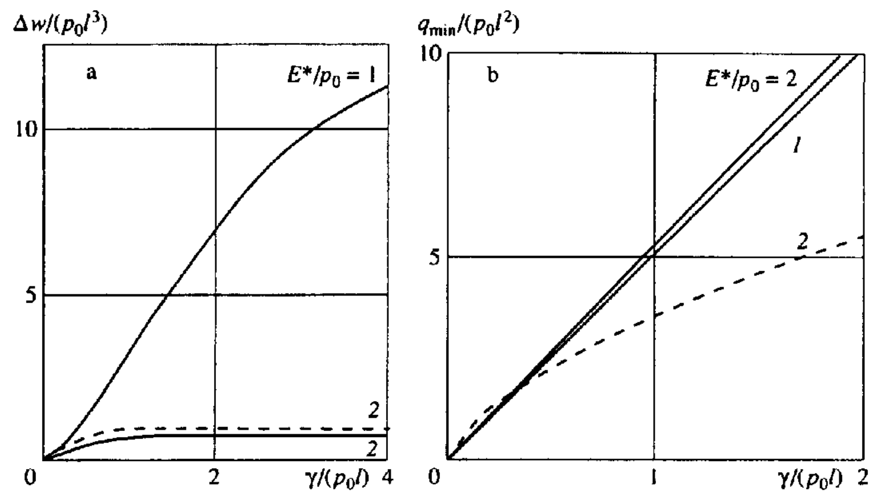

Fig. 5

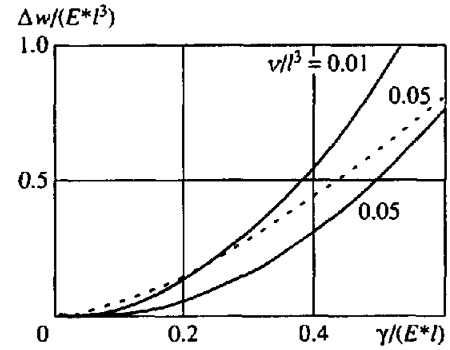

Fig. 6

$$Q = \frac{q}{\pi R \gamma}, \quad D = d\left(\frac{16E^{*2}}{9\pi^2\gamma^2 R}\right)^{1/3} \tag{3.2}$$

is described by a single parameter λ

$$\lambda = p_0\left(\frac{9R}{2\pi\gamma E^{*2}}\right)^{1/3}$$

which was first introduced by Tabor [11]. This parameter was also used in [1] when analysing adhesion in the contact of dry surfaces.

The work in the approach/separation cycle, represented in dimensionless form as

$$\Delta W = \Delta w\left(\frac{16E^2}{9\pi^5\gamma^5 R^4}\right)^{1/3}$$

is a function of one variable λ when $n = 1$. A graph of the function $\Delta W(\lambda)$ is shown in Fig. 7(a). Relations (2.3), (2.4) and (2.7), (2.9) which determine the solution of the problem for $n = 1$ become simplified and enable an analytical expression to be obtained for the function $\Delta W(\lambda)$

$$\Delta W = \frac{8704}{243\pi^3}\lambda^5 \quad \text{when} \quad 0 \leqslant \lambda < \lambda_0 = \left(\frac{9}{32}\right)^{1/3} \tag{3.3}$$

As $\lambda \to \infty$, the function ΔW tends to a constant value, which is obtained analytically and is approximately equal to $\Delta W_\infty \approx 1.80$. It has been shown [1] that the limiting case as $\lambda \to 0$ corresponds to the Deryagin–Muller–Toropov theory of adhesion, and the case when $\lambda \to \infty$ corresponds to the Johnson–Kendall–Roberts theory.

The pull-off force $q_{\min}$ can also be represented as a function of one parameter λ, which is similar to the result obtained previously [1].

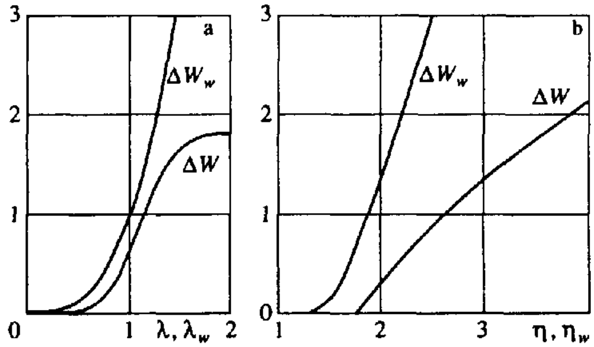

Fig. 7

In the case of the capillary adhesion between parabolic bodies, the relation between the dimensionless load Q and the dimensionless quantity D (Q and D are defined by (3.2)) is described by one dimensionless parameter

$$\eta = \frac{\gamma^{4/3} R^{5/3}}{v E^{*4/3}}$$

just as in the case of adhesion for dry surfaces. The dimensionless loss of energy ΔW specified by (3.3) in this case depends on one parameter η. A graph of the function $\Delta W(\eta)$ is shown in Fig. 7(b). The results indicate that the value of ΔW is non-zero only beginning from a certain value of η and increases without limit as η increases.

Note that for the bodies the shape of whose surfaces is described by a higher-degree polynomial ($n \geqslant 2$), one must use two dimensionless parameters to describe the dependence of the load on the change in the distance between the bodies (for example, the parameters E^*/p_0, $\gamma/(p_0 l)$ for the adhesion of dry surfaces and v/l^3, $\gamma/(E^* l)$ for capillary adhesion, which were used in Fig. 3).

4. USING THE WINKLER MODEL

The solution of the problem can be simplified considerably if we assume that the elastic properties of the bodies are described by the Winkler model. In this case, the normal displacement $u(r)$ of the surfaces is related to the applied pressure $p(r)$ by the formula

$$u(r) = kp(r), \qquad k = k_1 + k_2 \tag{4.1}$$

where k_1 and k_2 are constants describing the elastic properties of the bodies. Relation (4.1) is an approximate analog of relation (1.7).

Hence, relations (1.1)–(1.5) and (4.1) form a complete system of equations for the adhesion of dry surfaces, which relations (1.1)–(1.6) and (4.1) apply for capillary adhesion. For simplicity, we will consider the interaction between parabolic bodies, i.e., $n = 1$.

Taking (4.1) into account, from Eqs (1.1)–(1.5) we obtain the following relations which hold for both the adhesion of dry surfaces and capillary adhesion:

$$p(r) = -\frac{1}{k}\left(\frac{r^2}{2R} + d\right), \quad r \leqslant a; \quad u(r) = -kp_0, \quad a < r \leqslant b$$

$$a^2 = 2R(kp_0 - d), \quad b^2 = a^2 + 2R\gamma / p_0$$

$$q = -2\pi R\gamma + \pi(R/k)(d^2 - k^2 p_0^2) \tag{4.2}$$

when the surfaces are in contact ($h(0) = 0$);

$$u(r) = -kp_0, \quad 0 \leqslant r \leqslant b, \quad b^2 = -q/(\pi p_0)$$

$$q = -2\pi R\gamma + 2\pi R p_0 (d - kp_0) \tag{4.3}$$

when the surfaces are not in contact ($h(0) > 0$).

In the case of point contact, assuming $a = 0$ in (4.2), we have

$$q^* = -2\pi R\gamma, \quad d^* = kp_0. \tag{4.4}$$

Relations (4.2)–(4.4) define the solution of the problem in the case of adhesion between dry surfaces. To obtain the solution for capillary adhesion, we supplement these relations by the following expressions for the pressure in the fluid p_0, obtained from (1.6):

$$p_0 = \gamma(\pi R / v)^{1/2} \tag{4.5}$$

when the surfaces are in contact and

$$p_0^2 = -(q/v)[\gamma + q/(4\pi R)] \tag{4.6}$$

when the surfaces are not in contact.

Substituting these relations into (4.2)–(4.4), we obtain the solution for the case of capillary adhesion.

The solutions obtained define the relation between the load q and the quantity d. If we introduce the dimensionless variables Q (specified by the first relation of (3.2)) and $D = d(k\gamma)^{-1/2}$, the required relation will contain only one dimensionless parameter.

Thus, in the problem of adhesion between dry surfaces, using the parameter

$$\lambda_w = p_0(k/\gamma)^{1/2}$$

we can represent the last relations of (4.2) and (4.3) in the form

$$Q = \begin{cases} D_w^2 - 2 - \lambda_w^2, & D_w \leqslant \lambda_w \\ 2(\lambda_w D_w - \lambda_w^2 - 1), & D_w > \lambda_w \end{cases} \tag{4.7}$$

If $D_w \leqslant \lambda_w$, contact between the surfaces occurs. If $D_w > \lambda_w$, the surfaces are not in contact. The case when $D_w^* = \lambda_w$, $Q^* = -2$ corresponds to point contact.

For capillary adhesion, substituting (4.5) and (4.6) into the last relations of (4.2) and (4.3), respectively, and introducing the dimensionless parameter

$$\eta_w = \pi Rk\gamma/\upsilon$$

we obtain

$$D_w^2 = \begin{cases} Q + 2 + \eta_w, & D_w \leqslant \eta_w \\ -\dfrac{1}{\eta_w Q}\dfrac{(2 + Q - 2Q\eta - Q^2\eta/2)^2}{Q+4}, & D_w > \eta_w \end{cases} \tag{4.8}$$

If $D_w \leqslant \eta_w$, the surfaces will be in contact, while if $D_w > \eta_w$, the surfaces will not be in contact. The case when $D_w^* = \eta_w$, $Q^* = -2$ corresponds to point contact between the surfaces.

Relations (4.7) and (4.8) show that the dependence of the load Q on the quantity D_w is non-monotonic and unique for the adhesion of dry surfaces, whereas for capillary adhesion it is non-monotonic and non-unique. A comparison between these relations and the solutions corresponding to the exact formulation of the problem indicates that the Winkler model gives a relation for the load, which is qualitatively correct only for capillary adhesion. For the adhesion of dry surfaces, this relation is in qualitative agreement with the exact dependence only for the case of contact between the surfaces.

On the basis of the above relations between the load Q and the quantity D_w, we obtained relations for the dimensionless loss of energy in an approach/separation cycle

$$\Delta W_w = \frac{\Delta w}{\pi R}\left(\frac{1}{k\gamma^3}\right)^{1/2}$$

as a function of the parameters λ_w and η_w for the adhesion of dry surfaces and capillary adhesion, respectively. These relations are shown in Fig. 7. For the adhesion of dry surfaces, the function $\Delta W_w(\lambda_w)$ is obtained analytically

$$\Delta W_w = \begin{cases} \lambda_w^3, & \lambda_w \leqslant 1 \\ \dfrac{1}{3}\dfrac{(\lambda_w^2 - 1)(2\lambda_w^4 + 5\lambda_w^2 - 1)}{\lambda_w^3} + \dfrac{1}{\lambda_w}, & \lambda_w > 1 \end{cases}$$

This function is close to the function $\Delta W(\lambda)$ (Fig. 7a) corresponding to the exact formulation of the problem only for small λ_w.

In the case of capillary adhesion, the curve of $\Delta W_w(\eta_w)$ is qualitatively similar to the curve of $\Delta W(\eta)$ obtained for the exact formulation of the problem. The value of η_w, beginning from which the loss of energy ΔW_w becomes non-zero is $3\sqrt{3/4}$.

5. CONCLUSIONS

The interaction between elastic bodies has been investigated both for contact between the surfaces and for the case when the surfaces are not in contact. The surface energy of the bodies or of the fluid films covering these bodies is taken into account. The analysis enables us to draw the following conclusions.

1. For the adhesion of dry surfaces, the dependence of the load on the change in the distance between the bodies is non-unique for any values of the parameters of the problem. For capillary adhesion, this dependence is non-unique only for a certain range of variation of these parameters.

2. The loss of energy in an approach/separation cycle increases as the surface energy increases and the volume of fluid in the meniscus decreases. The loss of energy is greater for softer bodies (with smaller modulus of elasticity).

3. For parabolic bodies, the dependence of the load on the change in the distance between the bodies can be described by one dimensionless parameter. Accordingly, the dimensionless loss of energy in an approach/separation cycle is a function of one parameter for both the adhesion of dry surfaces and capillary adhesion. If the shape of the bodies is described by a power function of higher degree, then two dimensionless parameters are needed to describe these relations.

4. Using the Winkler model to describe the elastic properties of the bodies one obtains qualitatively correct results for capillary adhesion. For the adhesion of dry surfaces, these results are correct only for small values of the parameter λ.

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